From the motion of the planets to elementary particle physics

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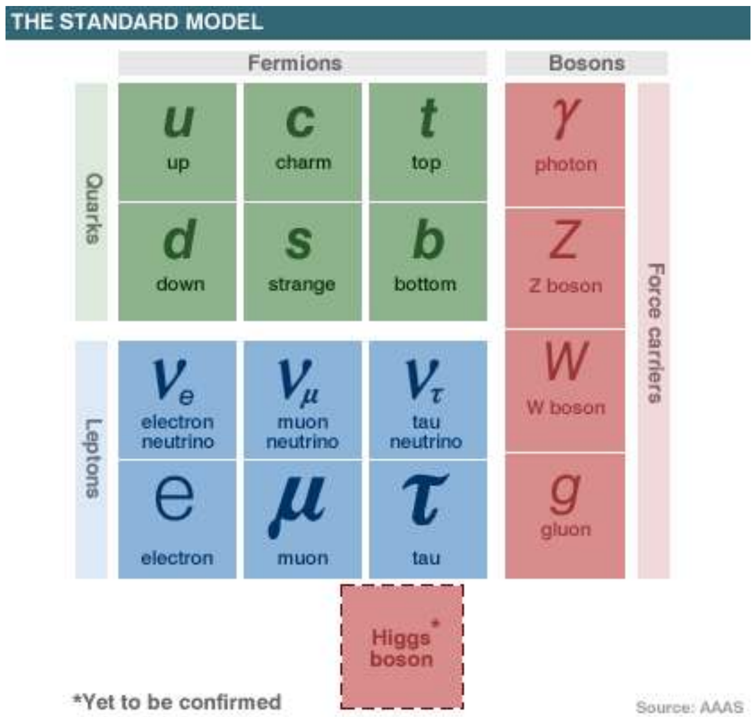
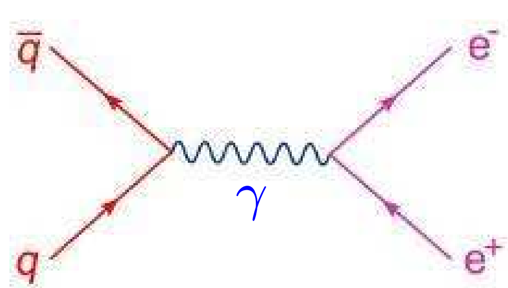
Standard model of elementary particles



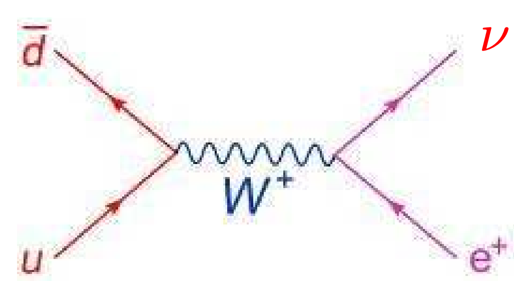
- Higgs boson: predicted by theorists in the 60's

- as of July 4th, 2012 : discovery by CMS and Atlas experiments

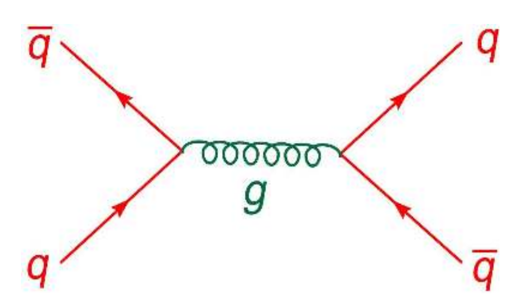
- matter built from spin 1/2 fermions
- forces (except gravity) carried by spin1 vector bosons



electromagnetism



weak



strong (QCD)

- non-Abelian gauge theory, gluon part:

$$\mathcal{L} = \frac{1}{4}\mathrm{Tr}\int F_{\mu\nu}F^{\mu\nu}\,, \qquad F^{\mu\nu} = \partial^\mu A^\nu - \partial^\nu A^\mu + ig[A^\mu, A^\nu]$$

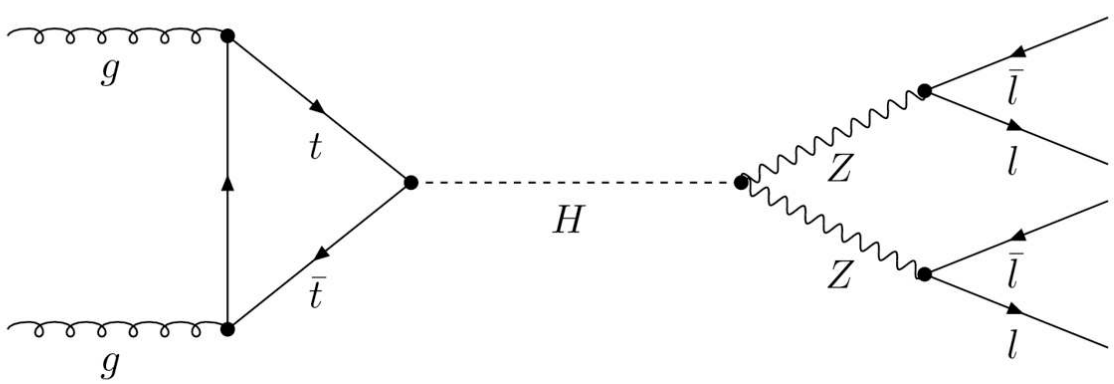
$$A^\mu = \sum_{a=1}^{N^2-1} A_a^\mu\, t_{ij}^a$$

gauge group SU(N), N=3

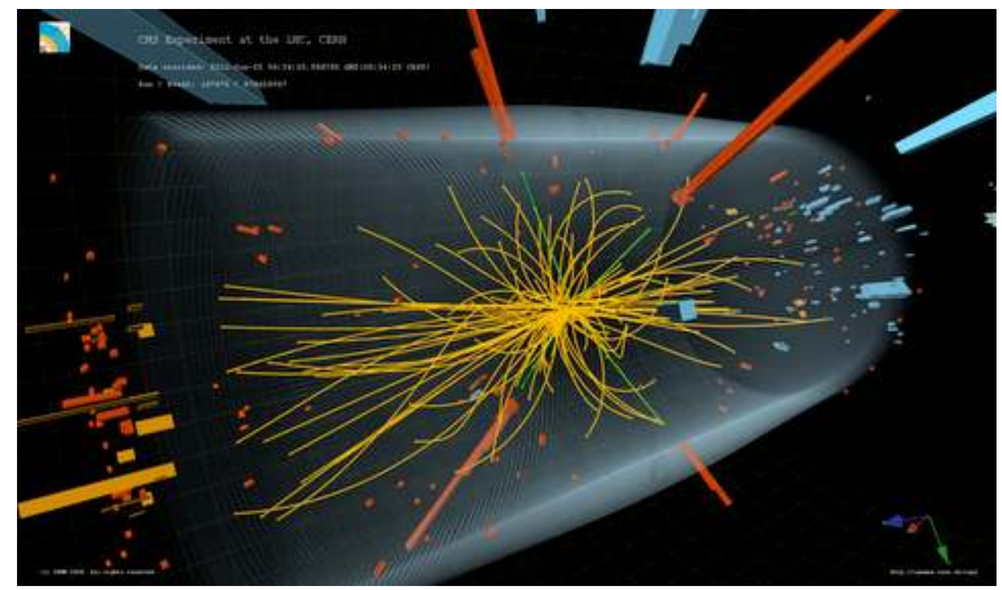
The Higgs at the LHC

- one of the decay modes of the Higgs boson

$$\text{gluon} + \text{gluon} \longrightarrow \text{Higgs} \longrightarrow e^{+} + e^{-} + \mu^{+} + \mu^{-}$$



Theory



Experiment

- feature of proton colliders: large QCD background

- identification of Higgs boson requires detailed understanding of many scattering processes

Challenges

- the Higgs boson was recently discovered
  - determine precisely its properties (mass, couplings to other particles)
  - detect new particles (e.g. hints for supersymmetry?)
- second run of the LHC
  - experimental precision will reach unprecedented levels
  - theory needs to match this precision

challenging experimenter’s wishlist

NNLO QCD and NLO EW Les Houches Wishlist

Wishlist part 1 - Higgs (V=W,Z)

Process	known	desired	motivation
H	d\sigma @ NNLO QCD d\sigma @ NLO EW finite quark mass effects @ NLO	d\sigma @ NNNLO QCD + NLO EW MC@NNLO finite quark mass effects @ NNLO	H branching ratios and couplings
H+j	d\sigma @ NNLO QCD (g only) d\sigma @ NLO EW	d\sigma @ NNLO QCD + NLO EW finite quark mass effects @ NLO	H p_T
H+2j	\sigma_tot(VBF) @ NNLO(DIS) QCD d\sigma(gg) @ NLO QCD d\sigma(VBF) @ NLO EW	d\sigma @ NNLO QCD + NLO EW	H couplings
H+V	d\sigma(V decays) @ NNLO QCD d\sigma @ NLO EW	with H→bb @ same accuracy	H couplings
t\bar tH	d\sigma(stable tops) @ NLO QCD	d\sigma(NWA top decays) @ NLO QCD + NLO EW	top Yukawa coupling
HH	d\sigma @ LO QCD finite quark mass effects d\sigma @ NLO QCD large m_t limit	d\sigma @ NLO QCD finite quark mass effects d\sigma @ NNLO QCD	Higgs self coupling

Wishlist part 2 - jets and heavy quarks

Process	known	desired	motivation
t\bar t	\sigma_tot @ NNLO QCD d\sigma(top decays) @ NLO QCD d\sigma(stable tops) @ NLO EW	d\sigma(top decays) @ NNLO QCD + NLO EW	precision top/QCD, gluon PDF effect of extra radiation at high rapidity top asymmetries
t\bar t+j	d\sigma(NWA top decays) @ NLO QCD	d\sigma(NWA top decays) @ NLO QCD + NLO EW	precision top/QCD, top asymmetries
single-top	d\sigma(NWA top decays) @ NLO QCD	d\sigma(NWA top decays) @ NNLO QCD (t channel)	precision top/QCD, V_tb
dijet	d\sigma @ NNLO QCD (g only) d\sigma @ NLO weak	d\sigma @ NNLO QCD + NLO EW	Obs.: incl. jets, dijet mass –> PDF fits (gluon at high x) –> alpha_s CMS x sections: http://arxiv.org/abs/1212.6660 [http://arxiv.org/abs/1212.6660]
3j	d\sigma @ NLO QCD	d\sigma @ NNLO QCD + NLO EW	Obs.: R3/2 or similar –> alpha_s at high pT dom. uncertainty: scales see http://arxiv.org/abs/1304.7498 [http://arxiv.org/abs/1304.7498] (CMS)
\gamma+j	d\sigma @ NLO QCD d\sigma @ NLO EW	d\sigma @ NNLO QCD + NLO EW	gluon PDF, \gamma+b for bottom PDF

Wishlist part 3 - EW gauge bosons (V=W,Z)

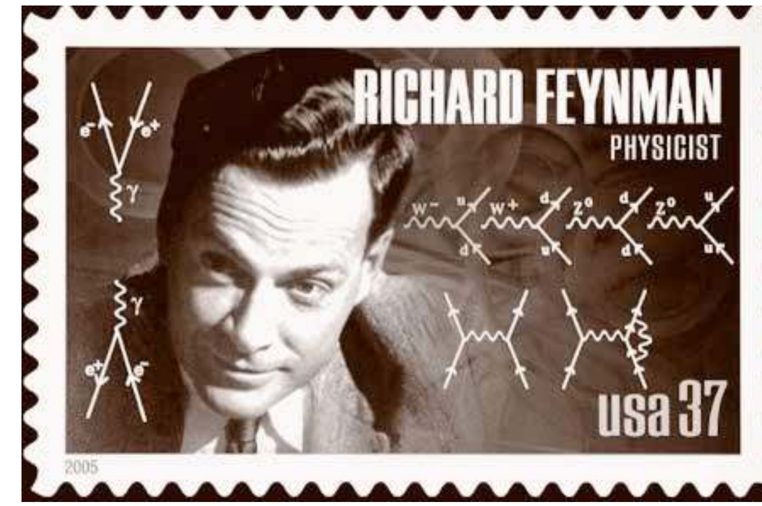
Process	known	desired	motivation
V	d\sigma(lept. V decay) @ NNLO QCD + EW	d\sigma(lept. V decay) @ NNNLO QCD + NLO EW MC@NNLO	precision EW, PDFs
V+j	d\sigma(lept. V decay) @ NLO QCD + EW	d\sigma(lept. V decay) @ NNLO QCD + NLO EW	Z+j for gluon PDF W+c for strange PDF
V+jj	d\sigma(lept. V decay) @ NLO QCD	d\sigma(lept. V decay) @ NNLO QCD + NLO EW	study of systematics of H+jj final state
VV'	d\sigma(V decays) @ NLO QCD d\sigma(stable V) @ NLO EW	d\sigma(V decays) @ NNLO QCD + NLO EW	bkg H → VV TGCs
gg → VV	d\sigma(V decays) @ LO	d\sigma(V decays) @ NLO QCD	bkg to H→VV
V\gamma	d\sigma(V decay) @ NLO QCD d\sigma(PA, V decay) @ NLO EW	d\sigma(V decay) @ NNLO QCD + NLO EW	TGCs
Vb\bar b	d\sigma(lept. V decay) @ NLO QCD massive b	d\sigma(lept. V decay) @ NNLO QCD massless b	bgk to VH(→bb)
VV'\gamma	d\sigma(V decays) @ NLO QCD	d\sigma(V decays) @ NLO QCD + NLO EW	QGCs
VV'V''	d\sigma(V decays) @ NLO QCD	d\sigma(V decays) @ NLO QCD + NLO EW	QGCs, EWSB
VV'+1jet	d\sigma(V decays) @ NLO QCD	d\sigma(V decays) @ NLO QCD + NLO EW	bkg to H, BSM searches
VV'+2j	d\sigma(V decays) @ NLO QCD	d\sigma(V decays) @ NLO QCD + NLO EW	QGCs, EWSB
\gamma\gamma	d\sigma @ NNLO QCD		bkg to H→\gamma\gamma

Scattering amplitudes

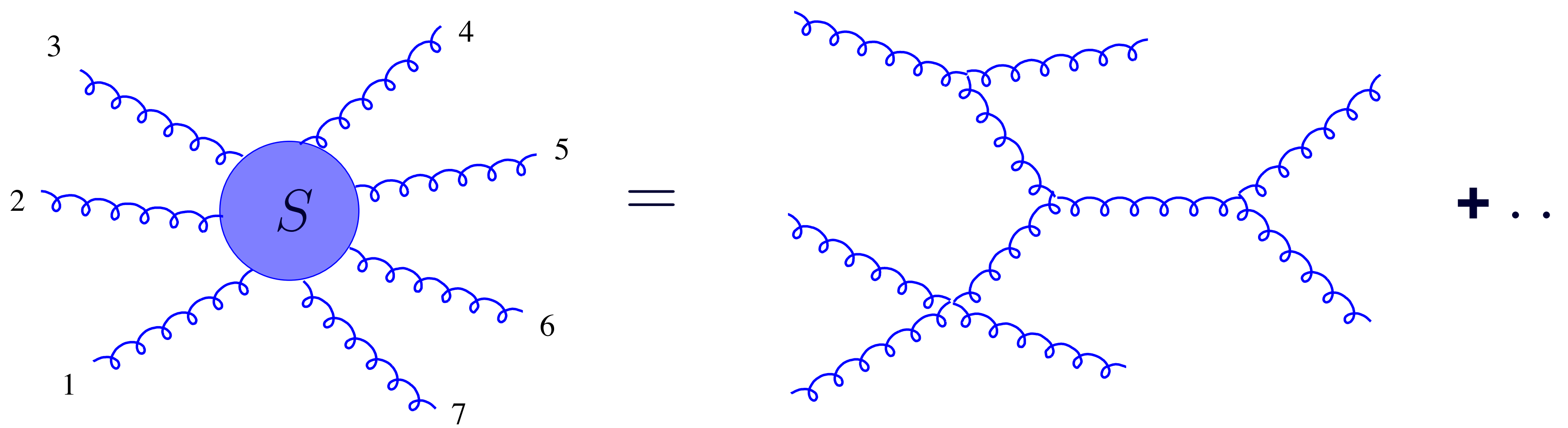
We know how to do this in principle:

(1) draw all Feynman diagrams

(2) compute them!



Often not so simple in practice! E.g. for gluon amplitudes:



number of external gluons	4	5	6	7	8	9	10
number of diagrams	4	25	220	2485	34300	559405	10525900

final results much simpler than intermediate steps!

understand why this is the case!

Outline

- Kepler problem and hydrogen atom are important classical and quantum mechanics problems that can be exactly solved

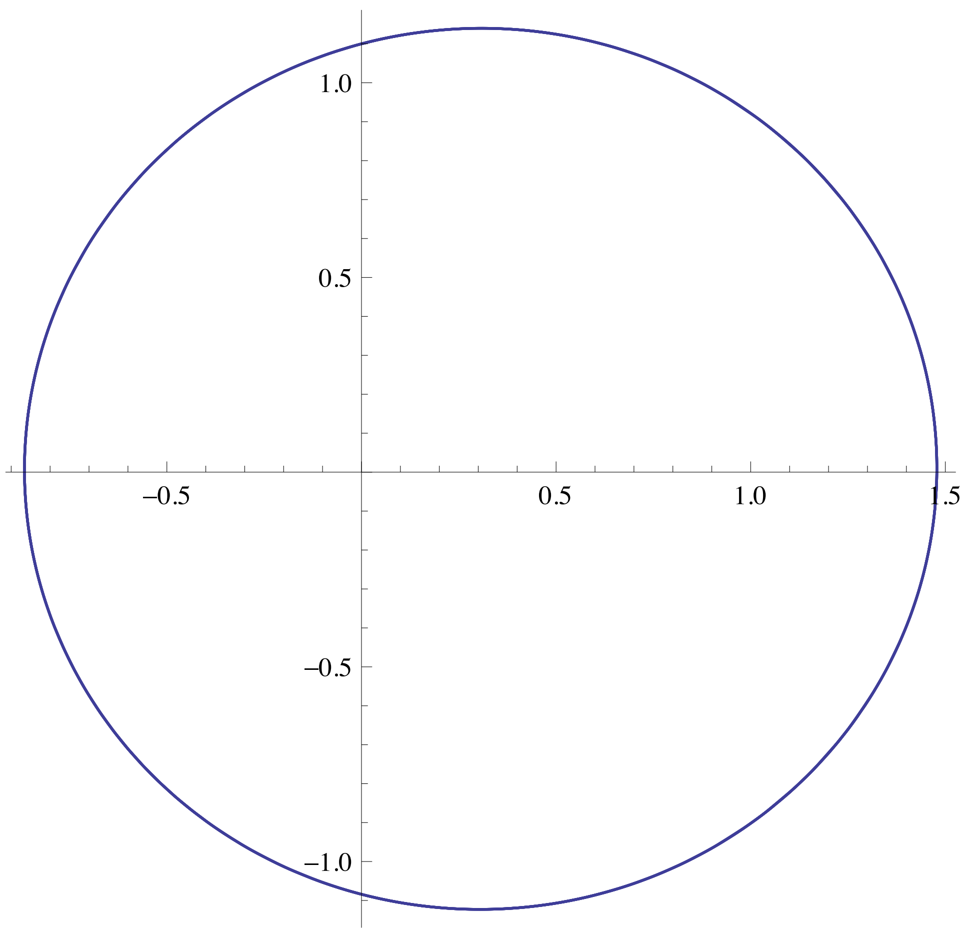
  they have a hidden symmetry

- is there a gauge theory analog of these systems?

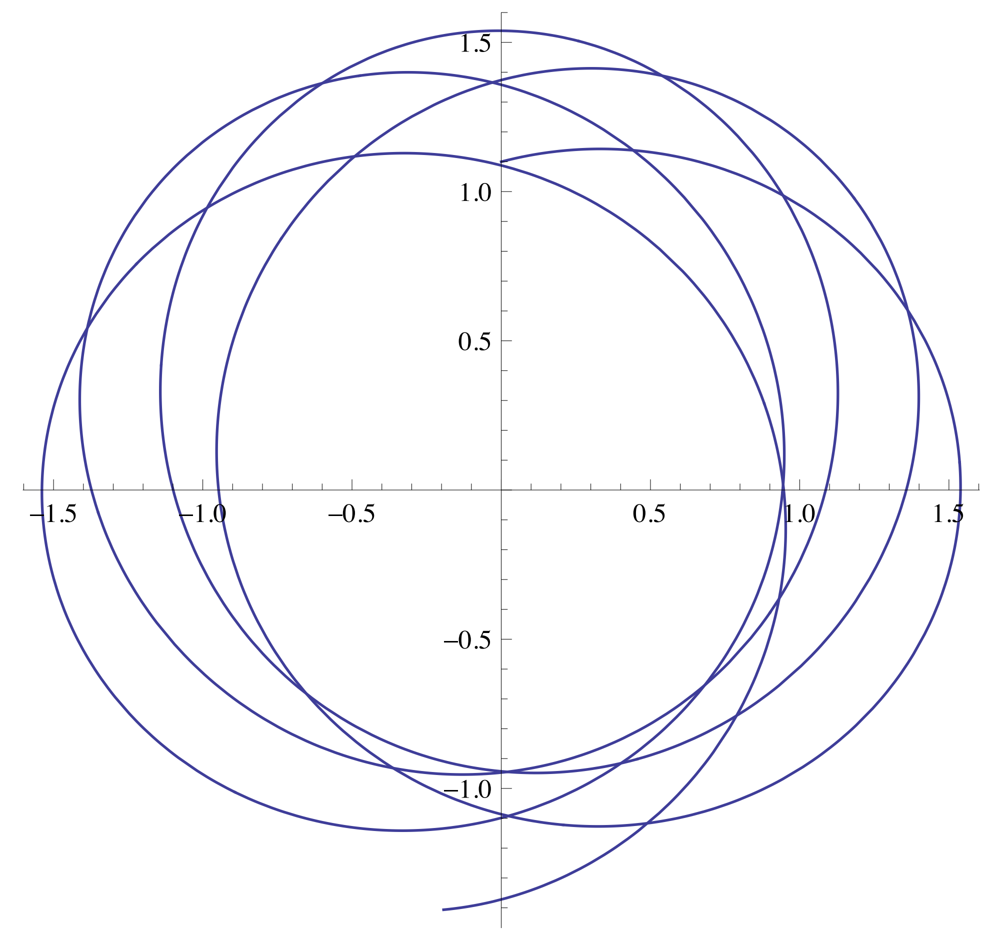
- how can we learn from this for understanding better QCD?

Kepler problem

$V = 1/r$



$V = 1/r^{0.9}$



- orbits do not precess
- conservation of Laplace-Runge-Lenz vector

$$\vec{A} = \frac{1}{2}\left(\vec{p}\times\vec{L} - \vec{L}\times\vec{p}\right) - \mu\frac{\lambda}{4\pi}\frac{\vec{x}}{|x|}$$

Hydrogen atom

- Hamiltonian $$H = \frac{1}{2m}p^2 - \frac{k}{r}$$

- hidden symmetry:
Laplace-Runge-Lenz-Pauli vector $$\vec{A} = \frac{1}{2}(\vec{p} \times \vec{L} - \vec{L} \times \vec{p}) - mk\frac{\vec{r}}{r}$$

- conserved quantity in quantum mechanics

$$[H, L_i] = 0 \qquad [H, A_i] = 0$$

$$[A_i, A_i] = -i\hbar\epsilon_{ijk}L_k\frac{2}{m}H$$

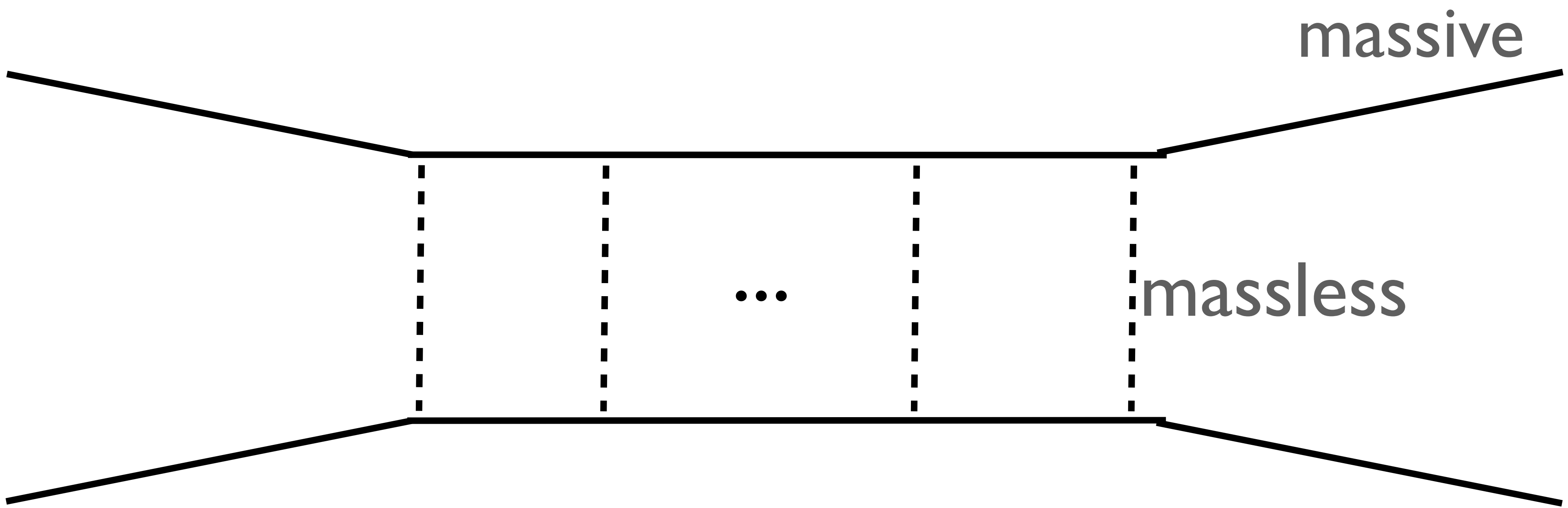
- operator algebra allows to find spectrum

$$E_n = -\frac{mk^2}{2\hbar^2}\frac{1}{n^2} \qquad n = 1, 2, \ldots$$

- degeneracy n^2

extension to a relativistic QFT

- Wick and Cutcosky considered the following model:



- This is the ladder approximation to $ep \to ep$, ignoring the spin of the photon

- In the non-relativistic limit, this reduces to the hydrogen Hamiltonian

SO(4) symmetry of Wick-Cutcosky model

- this model possesses an exact SO(4) symmetry, even away from the NR limit
- consider the Bethe-Salpether equation

$$\psi(p) = \int \frac{-4i\lambda\, m_1 m_3\, \psi(q)\, d^4q/(2\pi)^4}{(p-q)^2\left[(q-y_1)^2+m_1^2\right]\left[(y_3-q)^2+m_3^2\right]}$$

- not obvious in this form, but there is a conformal symmetry in momentum space
- spectrum only depends on cross-ratio

$$u = \frac{4m_1 m_3}{-s+(m_1-m_3)^2}$$

[Wick, 1954]

- apart from obvious symmetries, this contains the Laplace-Runge-Lenz vector! [Caron-Huot, JMH, 2014]

Beyond the ladder approximation

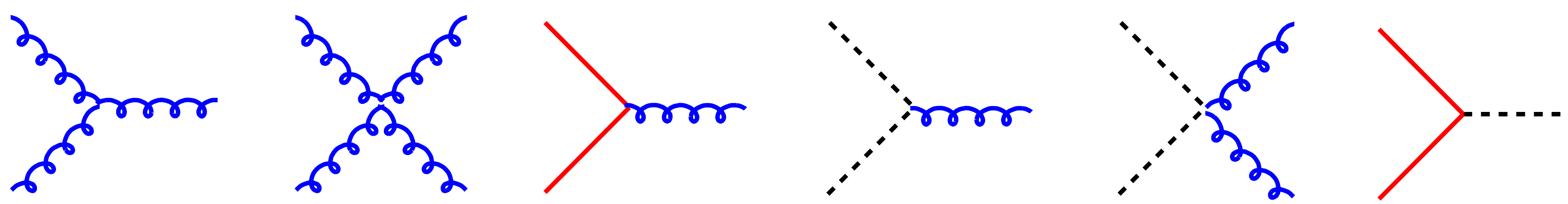
- Unfortunately, ladder approximation is not a consistent QFT
- e.g. misses multi-particle effects, and therefore has problems with unitarity
- is there a 4-dimensional QFT that has this hidden symmetry?

What is the simplest gauge theory?

- Maximally (super)symmetric gauge theory (without gravity)

massless spin-1 gluon	(= the same as in QCD)
4 massless spin-1/2 gluinos	(= cousin of the quarks)
6 massless spin-0 scalars	

- Interactions



- all proportional to same coupling g_{YM} and related by supersymmetry

- Many special features, e.g.:
  - Exactly scale-invariant for any coupling
  - Dual to a string theory (weak/strong coupling duality)

N=4 super Yang-Mills

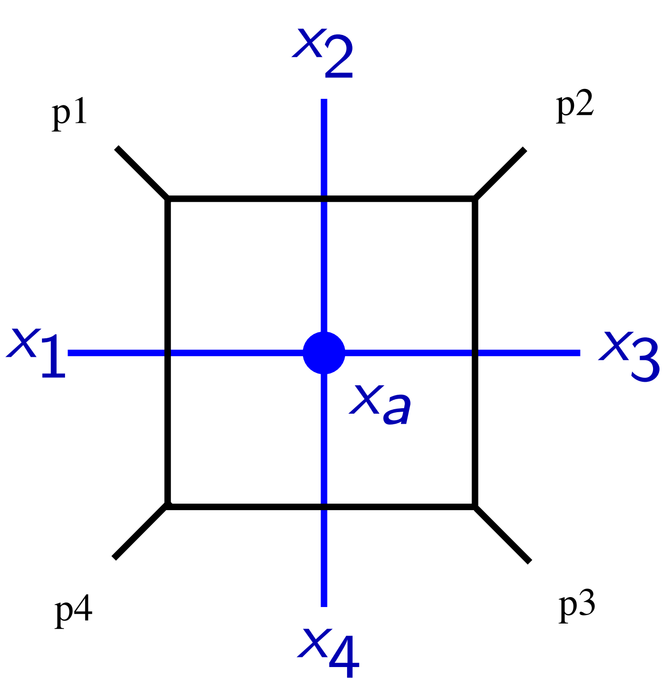
fast-forward from 1950's to 2000's

N=4 SYM has dual conformal symmetry

[Drummond, JMH, Smirnov, Sokatchev; Alday, Maldacena; Drummond, JMH, Korchemsky, Sokatchev; ...]

in massless sector:

$$\boxed{p_i^\mu = x_i^\mu - x_{i+1}^\mu}$$



$$= x_{13}^2 x_{24}^2 \int \frac{d^D x_a}{x_{1a}^2 x_{2a}^2 x_{3a}^2 x_{4a}^2}$$

invariant under SO(4,2) in dual space $\quad x^\mu \to x^\mu / x^2$

- this symmetry is at the heart of many developments

  - duality Wilson loops/scattering amplitudes

  - integrability of N=4 SYM theory

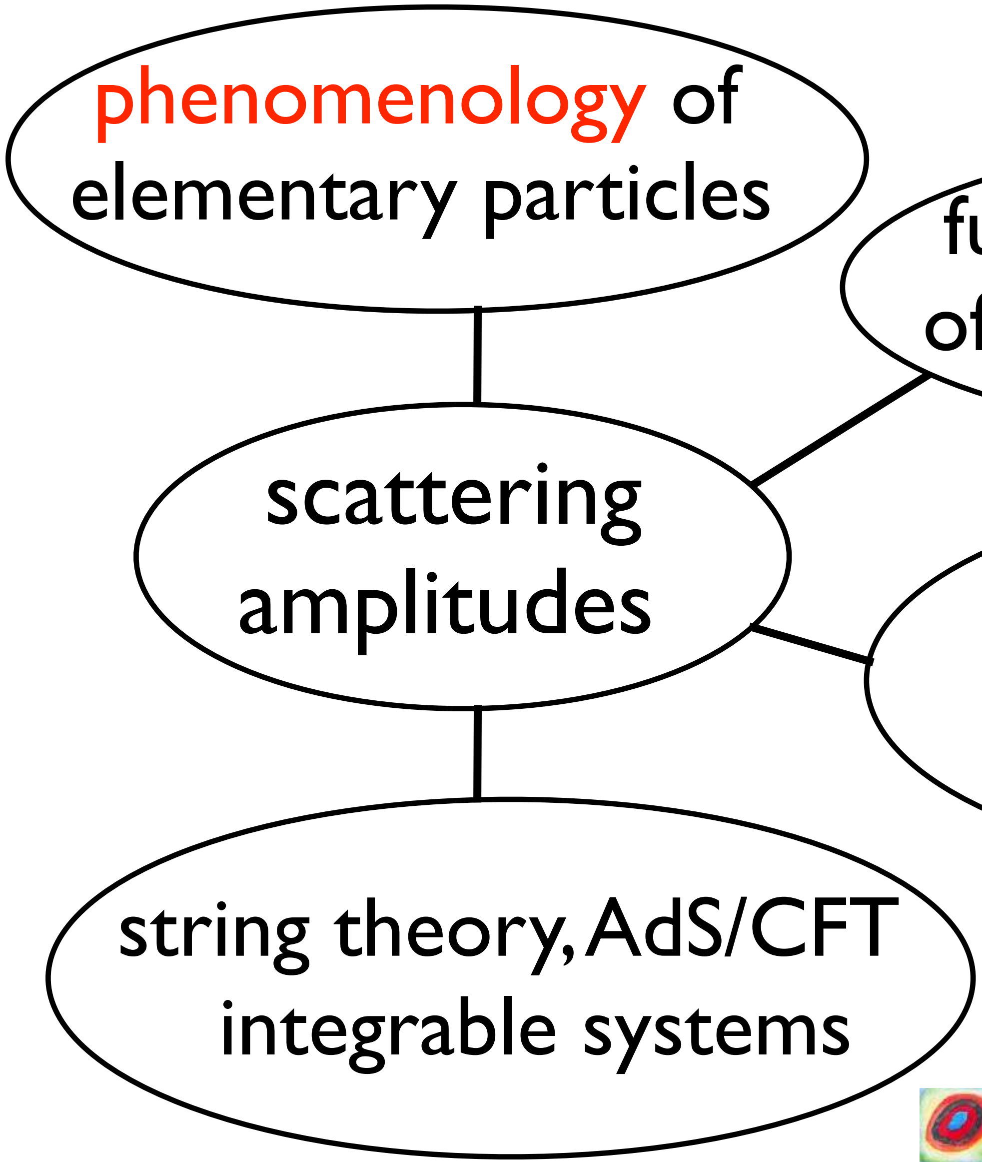
  - and other recent developments

- we have just argued that it is a natural generalization of the hydrogen atom's SO(4), itself inherited from the Kepler problem

Plan for remainder of the talk

- What surprises await us when studying scattering amplitudes in N=4 sYM?
- What can we learn for the difficult calculations that we need to perform in QCD?

Fruitful interplay of research fields



Meanwhile, in Hollywood…

- Sheldon Cooper working on these problems

duality between scattering amplitudes and Wilson loop

(in maximally supersymmetric Yang-Mills theory)

Reminder: Wilson lines/loops?

- phase factors (QM analog: Aharonov-Bohm effect)

$$W_C[x_1, x_2] = P e^{ig \int_C dx_\mu A^\mu}$$

P: path ordering

- required for gauge invariance of non-local objects

$\bar{\Psi}(x_2)$

$W_C[x_1, x_2]$

$\Psi(x_1)$

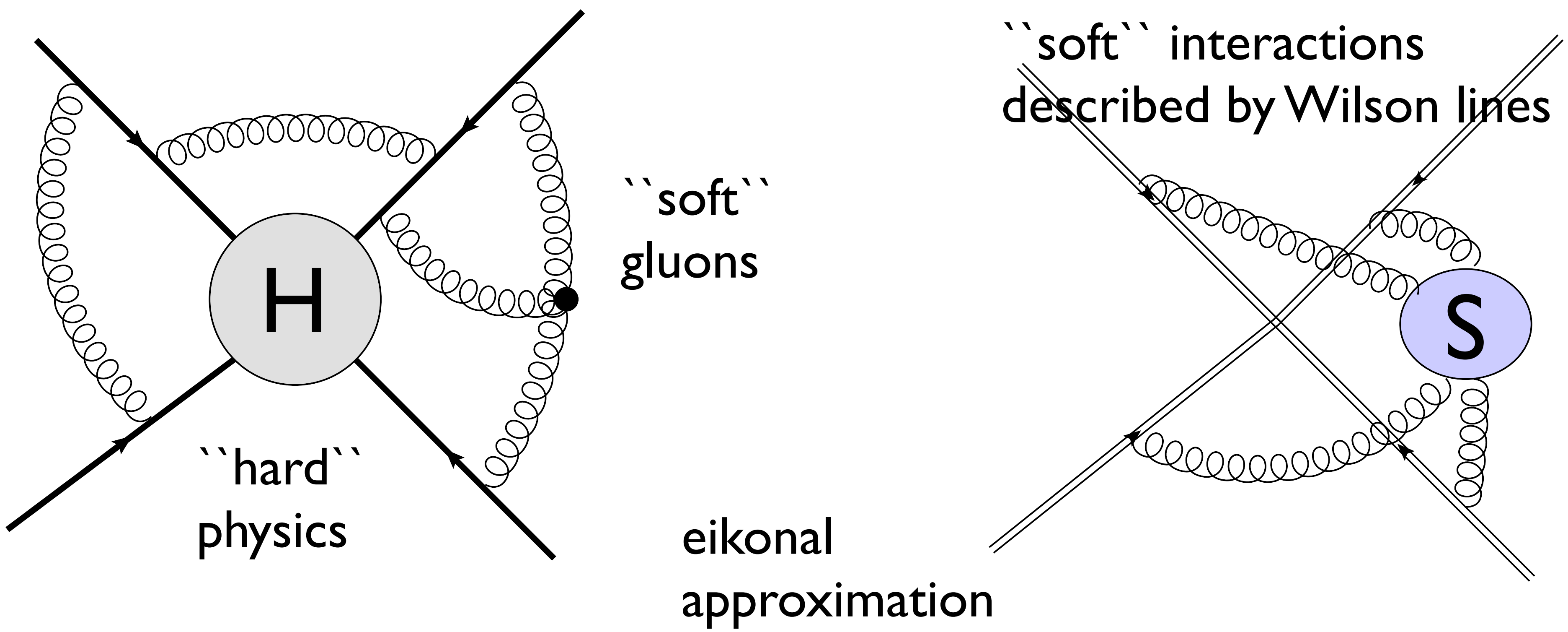
$$\mathcal{O} = \Psi(x_1) W_C[x_1, x_2] \bar{\Psi}(x_2)$$

- Wilson line $W_C[x_1, x_2]$ - loop for $x_1 = x_2$

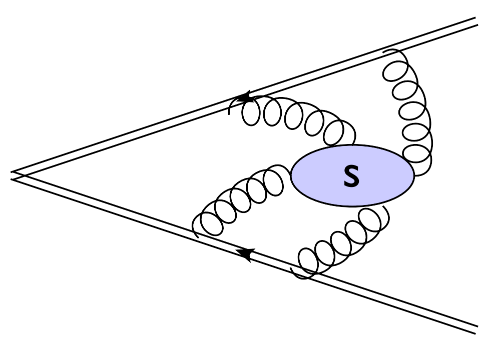
- related to many physical observables

Physical examples of Wilson lines

- infrared divergences of massive amplitudes



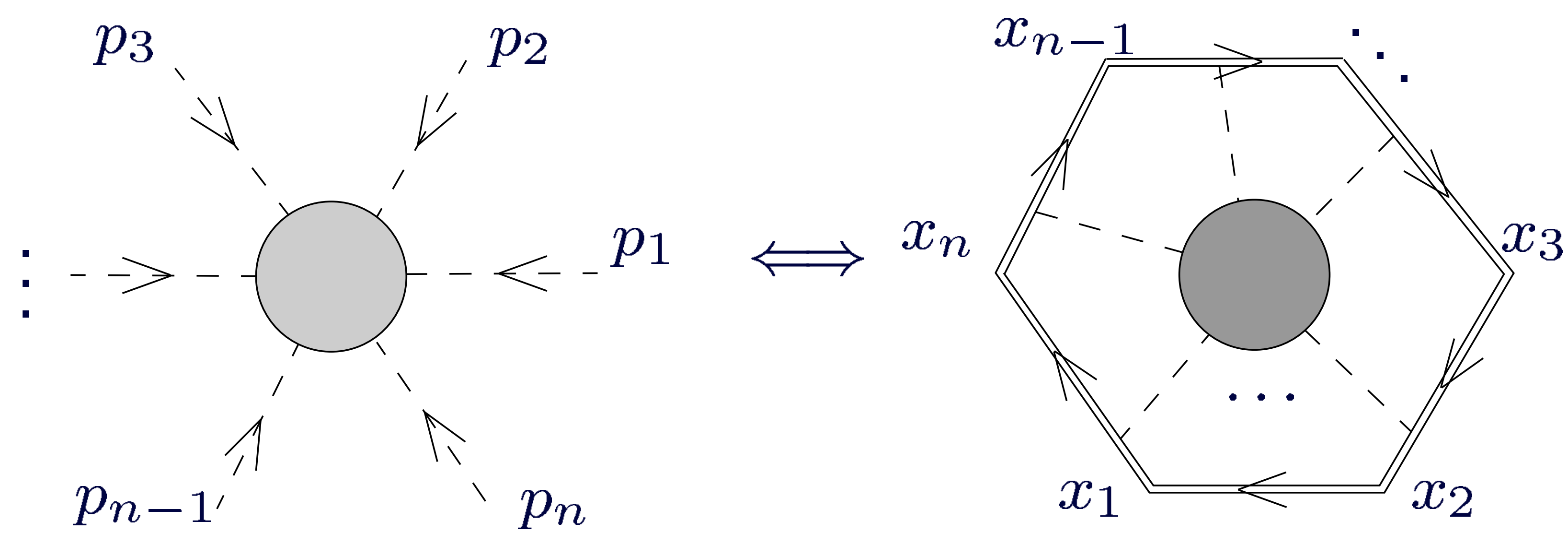
planar case: factorization into two-particle ``wedges``



$\Gamma_{\rm cusp}$

resummation of soft divergences $\qquad \mathcal{A} \sim e^{-|\log \mu_{IR}|\,\Gamma_{\rm cusp}}$

Duality between Wilson loops and scattering amplitudes



$$\boxed{x^\mu_{i+1} - x^\mu_i = p^\mu_i}$$

n-gluon scattering amplitude

$\mathcal{A}_n$

Wilson loop with n cusps $W[C_n]$

$$C_n = [x_1, x_2] \cup \ldots \cup [x_n, x_1]$$

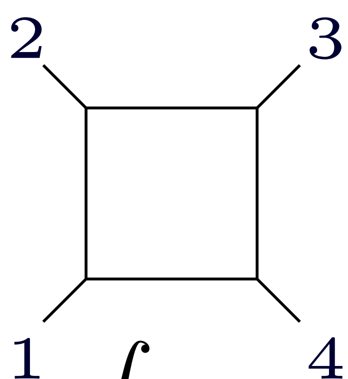
- up to non-planar corrections $O(1/N^2)$
- motivated by AdS/CFT correspondence
- many perturbative checks

Consequences of the duality

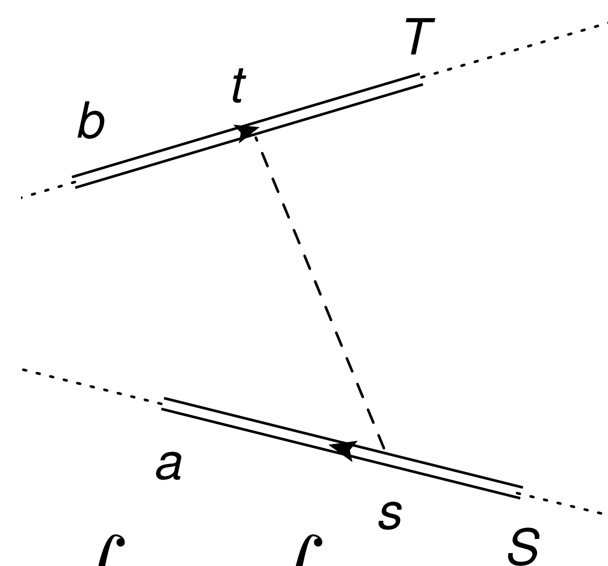
- collinear limits of scattering amplitudes $p_1 \,||\, p_2$
  $\longleftrightarrow$ contour deformation for Wilson loops

  described by (conformal) operator product expansion (OPE)

- relation between momentum space spacetime integrals and position space line integrals



$$\int d^4k \quad \longleftrightarrow \quad \int ds \int dt$$



- implies extra symmetries

  symmetries of Lagrangian
  act in momentum space and in dual coordinate space
  (on scattering amplitude) (on Wilson loop)

tree-level amplitudes and hidden symmetries

conformal symmetry

- angle preserving transformations

  Poincare: translations and rotations

  dilatations $D: \quad x^\mu \longrightarrow \lambda x^\mu$

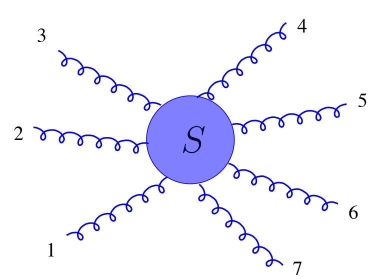
  special conformal transformations $K^\mu: \quad x^\mu \longrightarrow \dfrac{x^\mu - a^\mu x^2}{1 - 2a\cdot x + a^2 x^2}$

- typical property of massless theories or at high energy

- consequence of conformal symmetry

  generators of so(4,2): $J_i^a \qquad [J_i^a, J_j^b] = f^{abc}\delta_{ij} J_i^c$

  action on n-gluon amplitude: $J^a = \sum_{i=1}^n J_i^a$ (as for spin chain)

J^a 

$= 0$

hidden dual conformal symmetry

- hidden symmetry (as in hydrogen atom)

  dual coordinates: $p_i = x_i - x_{i+1}$

  two copies of (super)conformal symmetry

  (1) acting on momentum space

  (2) acting on dual coordinate space $K_\mu = i(x^2\partial_\mu - 2x_\mu x^\nu \partial_\nu)$

- mathematical structure

$$J^a = \sum_{i=1}^{n} J_i^a \qquad Q^c = f^{abc} \sum_{i<j}^{n} J_i^a J_j^b$$

  define Yangian algebra

- massless tree-level amplitudes have both symmetries!

$$J^a \mathcal{A}_n = 0 \qquad Q^a \mathcal{A}_n = 0$$

explains why they are so simple

more on trees

- analytic structure: recursion relations
  - Berends-Giele off-shell
  - BCFW on-shell [Britto, Cachazo, Feng + Witten]
- symmetries: Yangian invariance [Drummond, JMH, Plefka]
- twistor-string formulas [Witten][Spradlin, Volovich]
- new intriguing representations, scattering equations [Cachazo, He, Yuan]
- reformulation in terms of Grassmannian integral [Arkani-Hamed et al.]
- color-kinematics: BCJ relations [Bern, Carrasco, Johansson]
- ...
- important to study since many of these properties may also play a role at loop level!

new ideas for Feynman loop integrals

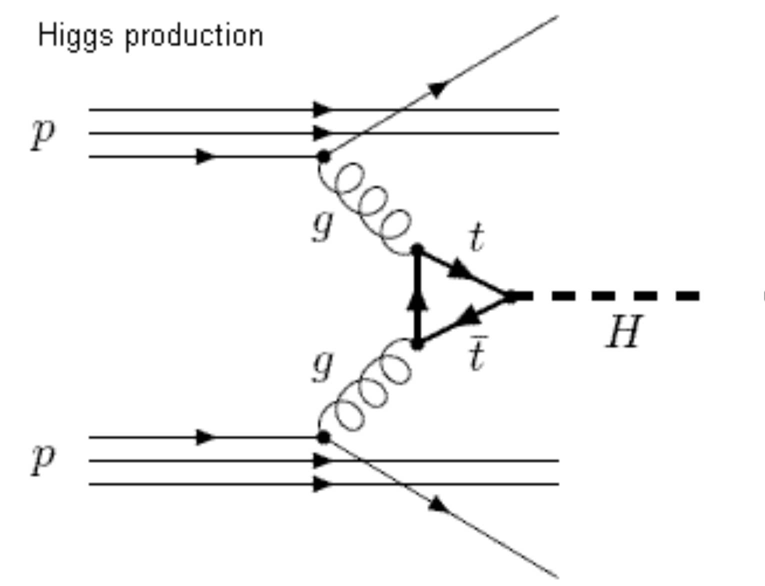
Scattering amplitudes at loop level

- What functions can are needed to describe them?
- Example: integral appearing in Higgs production

$$\int \frac{d^4k}{i\pi^2} \frac{1}{(m_t^2 - k^2)(m_t^2 - (k+p_1)^2)(m_t^2 - (k-p_2)^2)} =$$

$$= -\frac{1}{2s} \log^2 \left(\frac{\sqrt{1 - 4m_t^2/s} - 1}{\sqrt{1 - 4m_t^2/s} + 1} \right)$$

$$s = (p_1 + p_2)^2$$



- At one loop, only logarithm and dilogarithm needed

- More generally, special functions: multiple polylogarithms, elliptic polylogarithms…

From integrands to integrated functions

Main problem:

$$\mathcal{A}(\{x\}) \longrightarrow \int \prod_i dy_i I(\{x\},\{y\}) \xrightarrow{?} \mathcal{F}(\{x\})$$

$\mathcal{A}$ scattering amplitude for physical problem
function of momenta, masses

I integrand of Feynman integrals

$\mathcal{F}$ integrated functions

Key information in integrand

- Integrand is a rational function of momenta

- Its leading singularities carry a lot of information

[Cachazo; Cachazo, Skinner]

- Observation from N=4 sYM: integrals having constant leading singularities have very nice properties (uniform transcendental weight)

[Arkani-Hamed et al.]

- New idea:

[JMH]

(1) this property is not restricted to (finite integrals) N=4 sYM, easy to generalize to QCD in dimensional regularization

(2) use these integral as basis and compute them using differential equations

Key points of the method

[JMH, PRL 110 (2013) 25]

- differential equations for master integrals $\vec{f}$
- crucial: convenient basis choice simplifies differential equations
  $\longrightarrow$ makes solution trivial to obtain

- elegant description: Feynman integrals specified by:
  (1) set of 'letters' (related to singularities x_k)
  (2) set of constant matrices A_k

Example: one dimensionless variable x ; $D = 4 - 2\epsilon$

$$\partial_x \vec{f}(x;\epsilon) = \epsilon \sum_k \frac{A_k}{x - x_k} \vec{f}(x;\epsilon)$$

- expansion to any order in ϵ is linear algebra

answer: multiple polylogarithms of uniform weight ('transcendentality')

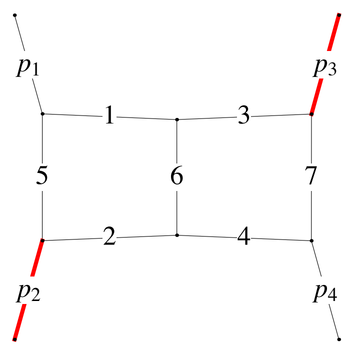
- asymptotic behavior $\vec{f}(x;\epsilon) \sim (x - x_k)^{\epsilon A_k} \vec{f}_0(\epsilon)$

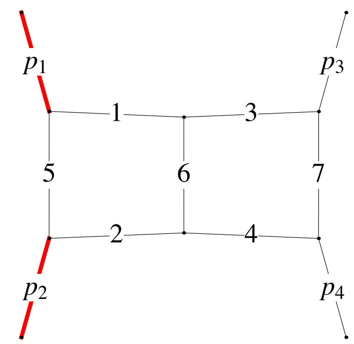
- natural extension to multi-variable case

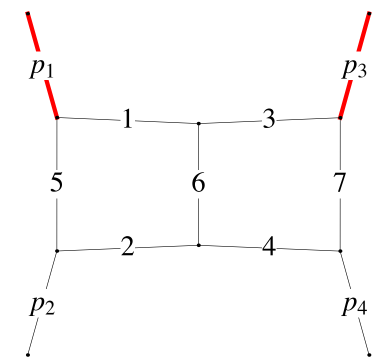
vector boson production $pp \longrightarrow VV$

- planar integral families

[JMH, Melnikov, Smirnov, 1402.7078]







- variables $\quad \dfrac{S}{M_3^2} = (1+x)(1+xy)\,, \quad \dfrac{T}{M_3^2} = -xz\,, \quad \dfrac{M_4^2}{M_3^2} = x^2 y$

  physical region $\quad 0 < x\,, \quad 0 < y < z < 1$

- differential equations

$$\boxed{\begin{aligned} d\vec{f}(x,y,z;\epsilon) &= \epsilon\, d\,\tilde{A}(x,y,z)\,\vec{f}(x,y,z;\epsilon) \\ \tilde{A} &= \sum_{i=1}^{15} \tilde{A}_{\alpha_i}\,\log(\alpha_i) \end{aligned}}$$

- alphabet

$$\alpha = \{x, y, z, 1+x, 1-y, 1-z, 1+xy, z-y, 1+y(1+x)-z, xy+z, \\ 1+x(1+y-z), 1+xz, 1+y-z, z+x(z-y)+xyz, z-y+yz+xyz\}.$$

- solution in terms of multiple polylogarithms

Summary Feynman integrals

- families of Feynman integrals described by iterated integrals

analytic answer specified by:

(1) "alphabet" for iterated integrals

(2) constant matrices provide rules to form words

- many applications for LHC physics, e.g.
  - amplitudes involving top quarks
  - vector boson production

- interesting physics and mathematics
  - singularities, asymptotic limits
  - hint for dualities: Knizhnik-Zamolodchikov equation
  - Hopf algebras, multiple zeta values
  - generalization to elliptic functions?

Summary

- N=4 super Yang-Mills is a QFT with a Laplace-Runge-Lenz symmetry

  - a lot of progress toward exactly solving the planar theory

  - uniqueness? are there other gauge theories with this symmetry?

  - is there a generalization beyond the planar limit?

- it is a close cousin to QCD

  - simplicity: e.g. tree-level amplitudes in QCD can be organized according to the hidden symmetry

  - suggests new techniques: e.g. new methods for computing generic Feynman loop integrals

- even after half a century, perturbative QCD still holds many (often pleasant) surprises!